Chapter 10

STRAIGHT LINES

10.1 Overview

10.1.1 *Slope of a line*

If θ is the angle made by a line with positive direction of x-axis in anticlockwise direction, then the value of tan θ is called the **slope of the line** and is denoted by m.

The slope of a line passing through points P(x_1, y_1) and Q (x_2, y_2) is given by

$$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$

10.1.2 ***Angle between two lines*** The angle θ between the two lines having slopes m_1 and m_2 is given by

$$\tan\theta = \pm \frac{(m_1 - m_2)}{1 + m_1 m_2}$$

If we take the acute angle between two lines, then $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

If the lines are parallel, then $m_1 = m_2$.
If the lines are perpendicular, then $m_1 m_2 = -1$.

10.1.3 ***Collinearity of three points*** If three points P (h, k), Q (x_1, y_1) and R (x_2, y_2) are such that slope of PQ = slope of QR, i.e., $\frac{y_1 - k}{x_1 - h} = \frac{y_2 - y_1}{x_2 - x_1}$

or $(h - x_1)(y_2 - y_1) = (k - y_1)(x_2 - x_1)$ then they are said to be collinear.

10.1.4 *Various forms of the equation of a line*

(i) If a line is at a distance a and parallel to x-axis, then the equation of the line is $y = \pm a$.

(ii) If a line is parallel to y-axis at a distance b from y-axis then its equation is $x = \pm b$

(iii) Point-slope form : The equation of a line having slope m and passing through the point (x_0, y_0) is given by $y - y_0 = m(x - x_0)$

(iv) Two-point-form : The equation of a line passing through two points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

(v) Slope intercept form : The equation of the line making an intercept c on y-axis and having slope m is given by

$$y = mx + c$$

Note that the value of c will be positive or negative as the intercept is made on the positive or negative side of the y-axis, respectively.

(vi) Intercept form : The equation of the line making intercepts a and b on x- and y-axis respectively is given by $\frac{x}{a} + \frac{y}{b} = 1$.

(vii) Normal form : Suppose a non-vertical line is known to us with following data:

(a) Length of the perpendicular (normal) p from origin to the line.

(b) Angle ω which normal makes with the positive direction of x-axis.

Then the equation of such a line is given by $x \cos \omega + y \sin \omega = p$

10.1.5 *General equation of a line*

Any equation of the form Ax + By + C = 0, where A and B are simultaneously not zero, is called the general equation of a line.

Different forms of Ax + By + C = 0

The general form of the line can be reduced to various forms as given below:

(i) Slope intercept form : If B $\neq$ 0, then Ax + By + C = 0 can be written as

$$y = \frac{-A}{B}x + \frac{-C}{B} \text{ or } y = mx + c, \text{ where } m = \frac{-A}{B} \text{ and } c = \frac{-C}{B}$$

If B = 0, then $x = \frac{-C}{A}$ which is a vertical line whose slope is not defined and x-intercept is $\frac{-C}{A}$.

(ii) Intercept form : If C $\neq$ 0, then Ax + By + C = 0 can be written as $\dfrac{x}{\frac{-C}{A}} + \dfrac{y}{\frac{-C}{B}}$ = 1 or $\frac{x}{a}+\frac{y}{b}=1$, where $a=\frac{-C}{A}$ and $b=\frac{-C}{B}$.

If C = 0, then Ax + By + C = 0 can be written as Ax + By = 0 which is a line passing through the origin and therefore has zero intercepts on the axes.

(iii) Normal Form : The normal form of the equation Ax + By + C = 0 is $x \cos\omega + y \sin\omega = p$ where,

$$\cos\omega = \pm\frac{A}{\sqrt{A^2+B^2}}, \sin\omega = \pm\frac{B}{\sqrt{A^2+B^2}} \text{ and } p = \pm\frac{C}{\sqrt{A^2+B^2}}.$$

Note: Proper choice of signs is to be made so that p should be always positive.

10.1.6 ***Distance of a point from a line*** The perpendicular distance (or simply distance) d of a point P (x_1, y_1) from the line Ax + By + C = 0 is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2+B^2}}$$

Distance between two parallel lines

The distance d between two parallel lines $y = mx + c_1$ and $y = mx + c_2$ is given by

$$d = \frac{|c_1 - c_2|}{\sqrt{1+m^2}}.$$

10.1.7 ***Locus and Equation of Locus*** The curve described by a point which moves under certain given condition is called its locus. To find the locus of a point P whose coordinates are (h, k), express the condition involving h and k. Eliminate variables if any and finally replace h by x and k by y to get the locus of P.

10.1.8 ***Intersection of two given lines*** Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are

(i) intersecting if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) parallel and distinct if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(iii) coincident if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Remarks

(i) The points (x_1, y_1) and (x_2, y_2) are on the same side of the line or on the opposite side of the line $ax + by + c = 0$, if $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of the same sign or of opposite signs respectively.

(ii) The condition that the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c = 0$ are perpendicular is $a_1a_2 + b_1b_2 = 0$.

(iii) The equation of any line through the point of intersection of two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is $a_1x + b_1y + c_1 + k\,(ax_2 + by_2 + c_2) = 0$. The value of k is determined from extra condition given in the problem.

10.2 Solved Examples

Short Answer Type

Example 1 Find the equation of a line which passes through the point (2, 3) and makes an angle of 30° with the positive direction of x-axis.

Solution Here the slope of the line is $m = \tan\theta = \tan 30° = \frac{1}{\sqrt{3}}$ and the given point is (2, 3). Therefore, using point slope formula of the equation of a line, we have

$$y - 3 = \frac{1}{\sqrt{3}}\,(x - 2) \qquad \text{or} \qquad x - \sqrt{3}\,y + (3\sqrt{3} - 2) = 0.$$

Example 2 Find the equation of the line where length of the perpendicular segment from the origin to the line is 4 and the inclination of the perpendicular segment with the positive direction of x-axis is 30°.

Solution The normal form of the equation of the line is $x \cos\omega + y \sin\omega = p$. Here $p = 4$ and $\omega = 30°$. Therefore, the equation of the line is

$$x \cos 30° + y \sin 30° = 4$$

$$x\frac{\sqrt{3}}{2} + y\,\frac{1}{2} = 4 \qquad \text{or} \qquad \sqrt{3}\,x + y = 8$$

Example 3 Prove that every straight line has an equation of the form $Ax + By + C = 0$, where A, B and C are constants.

Proof Given a straight line, either it cuts the y-axis, or is parallel to or coincident with it. We know that the equation of a line which cuts the y-axis (i.e., it has y-intercept) can be put in the form $y = mx + b$; further, if the line is parallel to or coincident with the y-axis, its equation is of the form $x = x_1$, where $x = 0$ in the case of coincidence. Both of these equations are of the form given in the problem and hence the proof.

Example 4 Find the equation of the straight line passing through (1, 2) and perpendicular to the line $x + y + 7 = 0$.

Solution Let m be the slope of the line whose equation is to be found out which is perpendicular to the line $x + y + 7 = 0$. The slope of the given line $y = (-1)\, x - 7$ is -1. Therefore, using the condition of perpendicularity of lines, we have $m \times (-1) = -1$ or $m = 1$ (Why?)

Hence, the required equation of the line is $y - 1 = (1)\,(x - 2)$ or $y - 1 = x - 2 \Rightarrow x - y - 1 = 0$.

Example 5 Find the distance between the lines $3x + 4y = 9$ and $6x + 8y = 15$.

Solution The equations of lines $3x + 4y = 9$ and $6x + 8y = 15$ may be rewritten as

$$3x + 4y - 9 = 0 \qquad \text{and} \qquad 3x + 4y - \frac{15}{2} = 0$$

Since, the slope of these lines are same and hence they are parallel to each other. Therefore, the distance between them is given by

$$\left|\frac{9 - \frac{15}{2}}{\sqrt{3^2 + 4^2}}\right| = \frac{3}{10}$$

Example 6 Show that the locus of the mid-point of the distance between the axes of the variable line $x \cos\alpha + y \sin\alpha = p$ is $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$ where p is a constant.

Solution Changing the given equation of the line into intercept form, we have $\frac{x}{\frac{p}{\cos\alpha}} + \frac{y}{\frac{p}{\sin\alpha}} = 1$ which gives the coordinates $\left(\frac{p}{\cos\alpha}, 0\right)$ and $\left(0, \frac{p}{\sin\alpha}\right)$, where the line intersects x-axis and y-axis, respectively.

Let (h, k) denote the mid-point of the line segment joining the points $\left(\frac{p}{\cos\alpha}, 0\right)$ and $\left(0, \frac{p}{\sin\alpha},\right)$

Then $h = \frac{p}{2\cos\alpha}$ and $k = \frac{p}{2\sin\alpha}$ (Why?)

This gives $\cos\alpha = \frac{p}{2h}$ and $\sin\alpha = \frac{p}{2k}$

Squaring and adding we get

$$\frac{p^2}{4h^2} + \frac{p^2}{4k^2} = 1 \quad \text{or} \quad \frac{1}{h^2} + \frac{1}{k^2} = \frac{4}{p^2}.$$

Therefore, the required locus is $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$.

Example 7 If the line joining two points A(2, 0) and B(3, 1) is rotated about A in anticlock wise direction through an angle of 15°. Find the equation of the line in new position.

Solution The slope of the line AB is $\frac{1-0}{3-2} = 1$ or $\tan 45°$ (Why?) (see Fig.). After rotation of the line through 15°, the slope of the line AC in new position is tan 60° = $\sqrt{3}$

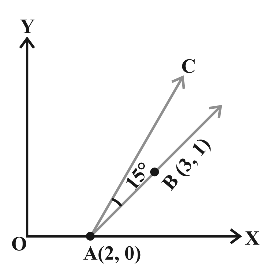

Fig. 10.1

Therefore, the equation of the new line AC is

$y - 0 = \sqrt{3}(x-2)$ or $y - \sqrt{3}x + 2\sqrt{3} = 0$

Long Answer Type

Example 8 If the slope of a line passing through the point A(3, 2) is $\frac{3}{4}$, then find points on the line which are 5 units away from the point A.

Solution Equation of the line passing through (3, 2) having slope $\frac{3}{4}$ is given by

$$y - 2 = \frac{3}{4}(x-3)$$

or $$4y - 3x + 1 = 0 \qquad (1)$$

Let (h, k) be the points on the line such that

$$(h-3)^2 + (k-2)^2 = 25 \qquad (2) \quad \text{(Why?)}$$

Also, we have

$$4k - 3h + 1 = 0 \qquad (3) \quad \text{(Why?)}$$

or $$k = \frac{3h-1}{4} \qquad (4)$$

Putting the value of k in (2) and on simplifying, we get

$$25h^2 - 150h - 175 = 0 \qquad \text{(How?)}$$

or $$h^2 - 6h - 7 = 0$$

or $$(h+1)(h-7) = 0 \Rightarrow h = -1, h = 7$$

Putting these values of k in (4), we get $k = -1$ and $k = 5$. Therefore, the coordinates of the required points are either $(-1, -1)$ or $(7, 5)$.

Example 9 Find the equation to the straight line passing through the point of intersection of the lines $5x - 6y - 1 = 0$ and $3x + 2y + 5 = 0$ and perpendicular to the line $3x - 5y + 11 = 0$.

Solution First we find the point of intersection of lines $5x - 6y - 1 = 0$ and $3x + 2y + 5 = 0$ which is $(-1, -1)$. Also the slope of the line $3x - 5y + 11 = 0$ is $\frac{3}{5}$. Therefore, the slope of the line perpendicular to this line is $\frac{-5}{3}$ (Why?). Hence, the equation of the required line is given by

$$y + 1 = \frac{-5}{3}(x + 1)$$

or $$5x + 3y + 8 = 0$$

Alternatively The equation of any line through the intersection of lines $5x - 6y - 1 = 0$ and $3x + 2y + 5 = 0$ is

$$5x - 6y - 1 + k(3x + 2y + 5) = 0 \qquad (1)$$

or Slope of this line is $\frac{-(5+3k)}{-6+2k}$

Also, slope of the line $3x - 5y + 11 = 0$ is $\frac{3}{5}$

Now, both are perpendicular

so $\frac{-(5+3k)}{-6+2k} \times \frac{3}{5} = -1$

or $$k = 45$$

Therefore, equation of required line in given by

$$5x - 6y - 1 + 45(3x + 2y + 5) = 0$$

or $$5x + 3y + 8 = 0$$

Example 10 A ray of light coming from the point (1, 2) is reflected at a point A on the x-axis and then passes through the point (5, 3). Find the coordinates of the point A.

Solution Let the incident ray strike x-axis at the point A whose coordinates be $(x, 0)$. From the figure, the slope of the reflected ray is given by

$$\tan\theta = \frac{3}{5 - x} \qquad (1)$$

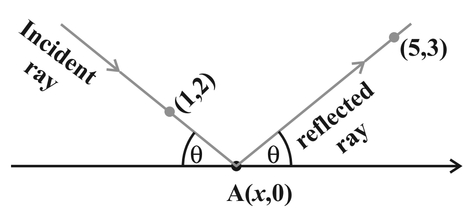

Fig. 10.2

Again, the slope of the incident ray is given by

$$\tan(\pi - \theta) = \frac{-2}{x-1} \qquad \text{(Why?)}$$

or

$$-\tan\theta = \frac{-2}{x-1} \qquad (2)$$

Solving (1) and (2), we get

$$\frac{3}{5-x} = \frac{2}{x-1} \quad \text{or} \quad x = \frac{13}{5}$$

Therefore, the required coordinates of the point A are $\left(\frac{13}{5}, 0\right)$.

Example 11 If one diagonal of a square is along the line $8x - 15y = 0$ and one of its vertex is at (1, 2), then find the equation of sides of the square passing through this vertex.

Solution Let ABCD be the given square and the coordinates of the vertex D be (1, 2). We are required to find the equations of its sides DC and AD.

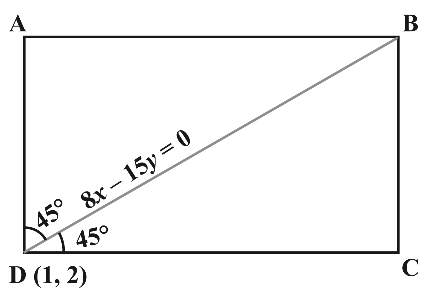

Fig. 10.3

Given that BD is along the line $8x - 15y = 0$, so its slope is $\frac{8}{15}$ (Why?). The angles made by BD with sides AD and DC is 45° (Why?). Let the slope of DC be m. Then

$$\tan 45° = \frac{m - \frac{8}{15}}{1 + \frac{8m}{15}} \qquad \text{(Why?)}$$

or $$15 + 8m = 15m - 8$$

or $$7m = 23, \text{ which gives } m = \frac{23}{7}$$

Therefore, the equation of the side DC is given by

$$y - 2 = \frac{23}{7}(x - 1) \text{ or } 23x - 7y - 9 = 0.$$

Similarly, the equation of another side AD is given by

$$y - 2 = \frac{-7}{23}(x - 1) \text{ or } 7x + 23y - 53 = 0.$$

Objective Type Questions

Each of the Examples 12 to 20 has four possible options out of which only one option is correct. Choose the correct option (M.C.Q.).

Example 12 The inclination of the line $x - y + 3 = 0$ with the positive direction of x-axis is

(A) 45° (B) 135° (C) – 45° (D) –135°

Solution (A) is the correct answer. The equation of the line $x - y + 3 = 0$ can be rewritten as $y = x + 3 \Rightarrow m = \tan\theta = 1$ and hence $\theta = 45°$.

Example 13 The two lines $ax + by = c$ and $a'x + b'y = c'$ are perpendicular if

(A) $aa' + bb' = 0$ (B) $ab' = ba'$

(C) $ab + a'b' = 0$ (D) $ab' + ba' = 0$

Solution (A) is correct answer. Slope of the line $ax + by = c$ is $\frac{-a}{b}$,

and the slope of the line $a'x + b'y = c'$ *is* $\frac{-a'}{b'}$. The lines are perpendicular if

$$\tan\theta = \frac{3}{5 - x} \quad (1)$$

$$\left(\frac{-a}{b}\right)\left(\frac{-a'}{b'}\right) = -1 \text{ or } aa' + bb' = 0 \quad \text{(Why?)}$$

Example 14 The equation of the line passing through (1, 2) and perpendicular to $x + y + 7 = 0$ is

(A) $y - x + 1 = 0$ (B) $y - x - 1 = 0$

(C) $y - x + 2 = 0$ (D) $y - x - 2 = 0$.

Solution (B) is the correct answer. Let the slope of the line be m. Then, its equation passing through (1, 2) is given by

$$y - 2 = m(x - 1) \qquad (1)$$

Again, this line is perpendicular to the given line $x + y + 7 = 0$ whose slope is -1 (Why?)

Therefore, we have $m(-1) = -1$

or $m = 1$

Hence, the required equation of the line is obtained by putting the value of m in (1), i.e.,

$$y - 2 = x - 1$$

or $y - x - 1 = 0$

Example 15 The distance of the point P (1, – 3) from the line $2y - 3x = 4$ is

(A) 13 (B) $\frac{7}{13}\sqrt{13}$ (C) $\sqrt{13}$ (D) None of these

Solution (A) is the correct answer. The distance of the point P (1, – 3) from the line $2y - 3x - 4 = 0$ is the length of perpendicular from the point to the line which is given by

$$\left|\frac{2(-3)-3-4}{\sqrt{13}}\right| = \sqrt{13}$$

Example 16 The coordinates of the foot of the perpendicular from the point (2, 3) on the line $x + y - 11 = 0$ are

(A) (–6, 5) (B) (5, 6) (C) (–5, 6) (D) (6, 5)

Solution (B) is the correct choice. Let (h, k) be the coordinates of the foot of the perpendicular from the point (2, 3) on the line $x + y - 11 = 0$. Then, the slope of the perpendicular line is $\frac{k-3}{h-2}$. Again the slope of the given line $x + y - 11 = 0$ is -1 (why?)

Using the condition of perpendicularity of lines, we have

$$\left(\frac{k-3}{h-2}\right)(-1) = -1 \qquad \text{(Why?)}$$

or $k - h = 1$ (1)

Since (h, k) lies on the given line, we have,

$$h + k - 11 = 0 \text{ or } h + k = 11 \qquad (2)$$

Solving (1) and (2), we get $h = 5$ and $k = 6$. Thus (5, 6) are the required coordinates of the foot of the perpendicular.

Example 17 The intercept cut off by a line from y-axis is twice than that from x-axis, and the line passes through the point (1, 2). The equation of the line is

(A) $2x + y = 4$ (B) $2x + y + 4 = 0$

(C) $2x - y = 4$ (D) $2x - y + 4 = 0$

Solution (A) is the correct choice. Let the line make intercept 'a' on x-axis. Then, it makes intercept '$2a$' on y-axis. Therefore, the equation of the line is given by

$$\frac{x}{a} + \frac{y}{2a} = 1$$

It passes through (1, 2), so, we have

$$\frac{1}{a} + \frac{2}{2a} = 1 \text{ or } a = 2$$

Therefore, the required equation of the line is given by

$$\frac{x}{2} + \frac{y}{4} = 1 \quad \text{or} \quad 2x + y = 4$$

Example 18 A line passes through P (1, 2) such that its intercept between the axes is bisected at P. The equation of the line is

(A) $x + 2y = 5$ (B) $x - y + 1 = 0$

(C) $x + y - 3 = 0$ (D) $2x + y - 4 = 0$

Solution The correct choice is (D). We know that the equation of a line making intercepts a and b with x-axis and y-axis, respectively, is given by

$$\frac{x}{a} + \frac{y}{b} = 1.$$

Here we have $$1 = \frac{a+0}{2} \text{ and } 2 = \frac{0+b}{2}, \qquad \text{(Why?)}$$

which give $a = 2$ and $b = 4$. Therefore, the required equation of the line is given by

$$\frac{x}{2}+\frac{y}{4}=1 \quad \text{or} \quad 2x+y-4=0$$

Example 19 The reflection of the point (4, – 13) about the line $5x + y + 6 = 0$ is

(A) (– 1, – 14) (B) (3, 4) (C) (0, 0) (D) (1, 2)

Solution The correct choice is (A). Let (h, k) be the point of reflection of the given point (4, – 13) about the line $5x + y + 6 = 0$. The mid-point of the line segment joining points (h, k) and (4, – 13) is given by

$$\left(\frac{h+4}{2}, \frac{k-13}{2}\right) \qquad \text{(Why?)}$$

This point lies on the given line, so we have

$$5\left(\frac{h+4}{2}\right)+\frac{k-13}{2}+6=0$$

or $\quad 5h + k + 19 = 0 \quad (1)$

Again the slope of the line joining points (h, k) and (4, –13) is given by $\frac{k+13}{h-4}$. This line is perpendicular to the given line and hence $(-5)\left(\frac{k+3}{h-4}\right)=-1$ (Why?)

This gives $\quad 5k + 65 = h - 4$

or $\quad h - 5k - 69 = 0 \quad (2)$

On solving (1) and (2), we get $h = -1$ and $k = -14$. Thus the point (–1, – 14) is the reflection of the given point.

Example 20 A point moves such that its distance from the point (4, 0) is half that of its distance from the line $x = 16$. The locus of the point is

(A) $3x^2 + 4y^2 = 192$ (B) $4x^2 + 3y^2 = 192$

(C) $x^2 + y^2 = 192$ (D) None of these

Solution The correct choice is (A). Let (h, k) be the coordinates of the moving point. Then, we have

$$\sqrt{(h-4)^2+k^2}=\frac{1}{2}\left(\frac{h-16}{\sqrt{1^2+0}}\right) \qquad \text{(Why?)}$$

$$\Rightarrow \quad (h-4)^2 + k^2 = \frac{1}{4}(h-16)^2$$

$$4(h^2 - 8h + 16 + k^2) = h^2 - 32h + 256$$

or $$3h^2 + 4k^2 = 192$$

Hence, the required locus is given by $3x^2 + 4y^2 = 192$

10.3 EXERCISE

Short Answer Type Questions

1. Find the equation of the straight line which passes through the point (1, – 2) and cuts off equal intercepts from axes.
2. Find the equation of the line passing through the point (5, 2) and perpendicular to the line joining the points (2, 3) and (3, – 1).
3. Find the angle between the lines $y = (2-\sqrt{3})(x+5)$ and $y = (2+\sqrt{3})(x-7)$.
4. Find the equation of the lines which passes through the point (3, 4) and cuts off intercepts from the coordinate axes such that their sum is 14.
5. Find the points on the line $x + y = 4$ which lie at a unit distance from the line $4x + 3y = 10$.
6. Show that the tangent of an angle between the lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{a}-\frac{y}{b}=1$ is $\frac{2ab}{a^2-b^2}$.
7. Find the equation of lines passing through (1, 2) and making angle 30° with y-axis.
8. Find the equation of the line passing through the point of intersection of $2x + y = 5$ and $x + 3y + 8 = 0$ and parallel to the line $3x + 4y = 7$.
9. For what values of a and b the intercepts cut off on the coordinate axes by the line $ax + by + 8 = 0$ are equal in length but opposite in signs to those cut off by the line $2x - 3y + 6 = 0$ on the axes.
10. If the intercept of a line between the coordinate axes is divided by the point (–5, 4) in the ratio 1 : 2, then find the equation of the line.
11. Find the equation of a straight line on which length of perpendicular from the origin is four units and the line makes an angle of 120° with the positive direction of x-axis.

[**Hint**: Use normal form, here ω =30°.]

12. Find the equation of one of the sides of an isosceles right angled triangle whose hypotenuse is given by $3x + 4y = 4$ and the opposite vertex of the hypotenuse is (2, 2).

Long Answer Type

13. If the equation of the base of an equilateral triangle is $x + y = 2$ and the vertex is (2, – 1), then find the length of the side of the triangle.

[**Hint**: Find length of perpendicular (p) from (2, – 1) to the line and use p = l sin 60°, where l is the length of side of the triangle].

14. A variable line passes through a fixed point P. The algebraic sum of the perpendiculars drawn from the points (2, 0), (0, 2) and (1, 1) on the line is zero. Find the coordinates of the point P.

[**Hint:** Let the slope of the line be m. Then the equation of the line passing through the fixed point P (x_1, y_1) is $y - y_1 = m(x - x_1)$. Taking the algebraic sum of perpendicular distances equal to zero, we get $y - 1 = m(x - 1)$. Thus (x_1, y_1) is (1, 1).]

15. In what direction should a line be drawn through the point (1, 2) so that its point of intersection with the line $x + y = 4$ is at a distance $\frac{\sqrt{6}}{3}$ from the given point.

16. A straight line moves so that the sum of the reciprocals of its intercepts made on axes is constant. Show that the line passes through a fixed point.

[**Hint:** $\frac{x}{a}+\frac{y}{b}=1$ where $\frac{1}{a}+\frac{1}{b}=$ constant = $\frac{1}{k}$ (say). This implies that $\frac{k}{a}+\frac{k}{b}=1 \Rightarrow$ line passes through the fixed point (k, k).]

17. Find the equation of the line which passes through the point (– 4, 3) and the portion of the line intercepted between the axes is divided internally in the ratio 5 : 3 by this point.

18. Find the equations of the lines through the point of intersection of the lines $x - y + 1 = 0$ and $2x - 3y + 5 = 0$ and whose distance from the point (3, 2) is $\frac{7}{5}$.

19. If the sum of the distances of a moving point in a plane from the axes is 1, then find the locus of the point. [**Hint**: Given that $|x| + |y| = 1$, which gives four sides of a square.]

20. P_1, P_2 are points on either of the two lines $y - \sqrt{3}\,|x| = 2$ at a distance of 5 units from their point of intersection. Find the coordinates of the foot of perpendiculars drawn from P_1, P_2 on the bisector of the angle between the given lines.

[**Hint**: Lines are $y = \sqrt{3}x + 2$ and $y = -\sqrt{3}\,x + 2$ according as $x \geq 0$ or $x < 0$. y-axis is the bisector of the angles between the lines. P_1, P_2 are the points on these lines at a distance of 5 units from the point of intersection of these lines which have a point on y-axis as common foot of perpendiculars from these points. The y-coordinate of the foot of the perpendicular is given by $2 + 5\cos 30°$.]

21. If p is the length of perpendicular from the origin on the line $\frac{x}{a} + \frac{y}{b} = 1$ and a^2, p^2, b^2 are in A.P, then show that $a^4 + b^4 = 0$.

Objective Type Questions

Choose the correct answer from the given four options in Exercises 22 to 41

22. A line cutting off intercept – 3 from the y-axis and the tengent at angle to the x-axis is $\frac{3}{5}$, its equation is

(A) $5y - 3x + 15 = 0$ (B) $3y - 5x + 15 = 0$

(C) $5y - 3x - 15 = 0$ (D) None of these

23. Slope of a line which cuts off intercepts of equal lengths on the axes is

(A) -1 (B) -0

(C) 2 (D) $\sqrt{3}$

24. The equation of the straight line passing through the point (3, 2) and perpendicular to the line $y = x$ is

(A) $x - y = 5$ (B) $x + y = 5$

(C) $x + y = 1$ (D) $x - y = 1$

25. The equation of the line passing through the point (1, 2) and perpendicular to the line $x + y + 1 = 0$ is

(A) $y - x + 1 = 0$ (B) $y - x - 1 = 0$

(C) $y - x + 2 = 0$ (D) $y - x - 2 = 0$

26. The tangent of angle between the lines whose intercepts on the axes are a, $-b$ and b, $-a$, respectively, is

(A) $\dfrac{a^2-b^2}{ab}$ (B) $\dfrac{b^2-a^2}{2}$

(C) $\dfrac{b^2-a^2}{2ab}$ (D) None of these

27. If the line $\dfrac{x}{a}+\dfrac{y}{b}=1$ passes through the points (2, –3) and (4, –5), then (a, b) is

(A) (1, 1) (B) (– 1, 1) (C) (1, – 1) (D) (– 1, –1)

28. The distance of the point of intersection of the lines $2x - 3y + 5 = 0$ and $3x + 4y = 0$ from the line $5x - 2y = 0$ is

(A) $\dfrac{130}{17\sqrt{29}}$ (B) $\dfrac{13}{7\sqrt{29}}$ (C) $\dfrac{130}{7}$ (D) None of these

29. The equations of the lines which pass through the point (3, –2) and are inclined at 60° to the line $\sqrt{3}\ x + y = 1$ is

(A) $y + 2 = 0,\ \sqrt{3}x - y - 2 - 3\sqrt{3} = 0$

(B) $x - 2 = 0,\ \sqrt{3}\ x - y + 2 + 3\sqrt{3} = 0$

(C) $\sqrt{3}\ x - y - 2 - 3\sqrt{3} = 0$

(D) None of these

30. The equations of the lines passing through the point (1, 0) and at a distance $\dfrac{\sqrt{3}}{2}$ from the origin, are

(A) $\sqrt{3}x + y - \sqrt{3} = 0,\ \sqrt{3}x - y - \sqrt{3} = 0$

(B) $\sqrt{3}x + y + \sqrt{3} = 0,\ \sqrt{3}x - y + \sqrt{3} = 0$

(C) $x + \sqrt{3}\ y - \sqrt{3} = 0,\ x - \sqrt{3}\ y - \sqrt{3} = 0$

(D) None of these.

31. The distance between the lines $y = mx + c_1$ and $y = mx + c_2$ is

(A) $\dfrac{c_1 - c_2}{\sqrt{m^2+1}}$ (B) $\dfrac{|c_1 - c_2|}{\sqrt{1+m^2}}$ (C) $\dfrac{c_2 - c_1}{\sqrt{1+m^2}}$ (D) 0

32. The coordinates of the foot of perpendiculars from the point (2, 3) on the line $y = 3x + 4$ is given by

(A) $\left(\frac{37}{10}, \frac{-1}{10}\right)$ (B) $\left(\frac{-1}{10}, \frac{37}{10}\right)$ (C) $\left(\frac{10}{37}, -10\right)$ (D) $\left(\frac{2}{3}, -\frac{1}{3}\right)$

33. If the coordinates of the middle point of the portion of a line intercepted between the coordinate axes is (3, 2), then the equation of the line will be

(A) $2x + 3y = 12$ (B) $3x + 2y = 12$ (C) $4x - 3y = 6$ (D) $5x - 2y = 10$

34. Equation of the line passing through (1, 2) and parallel to the line $y = 3x - 1$ is

(A) $y + 2 = x + 1$ (B) $y + 2 = 3(x + 1)$

(C) $y - 2 = 3(x - 1)$ (D) $y - 2 = x - 1$

35. Equations of diagonals of the square formed by the lines $x = 0$, $y = 0$, $x = 1$ and $y = 1$ are

(A) $y = x$, $y + x = 1$ (B) $y = x$, $x + y = 2$

(C) $2y = x$, $y + x = \frac{1}{3}$ (D) $y = 2x$, $y + 2x = 1$

36. For specifying a straight line, how many geometrical parameters should be known?

(A) 1 (B) 2 (C) 4 (D) 3

37. The point (4, 1) undergoes the following two successive transformations :

(i) Reflection about the line $y = x$

(ii) Translation through a distance 2 units along the positive x-axis

Then the final coordinates of the point are

(A) (4, 3) (B) (3, 4) (C) (1, 4) (D) $\left(\frac{7}{2}, \frac{7}{2}\right)$

38. A point equidistant from the lines $4x + 3y + 10 = 0$, $5x - 12y + 26 = 0$ and $7x + 24y - 50 = 0$ is

(A) (1, –1) (B) (1, 1) (C) (0, 0) (D) (0, 1)

39. A line passes through (2, 2) and is perpendicular to the line $3x + y = 3$. Its y-intercept is

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) 1 (D) $\frac{4}{3}$

40. The ratio in which the line $3x + 4y + 2 = 0$ divides the distance between the lines $3x + 4y + 5 = 0$ and $3x + 4y - 5 = 0$ is

(A) 1 : 2 (B) 3 : 7 (C) 2 : 3 (D) 2 : 5

41. One vertex of the equilateral triangle with centroid at the origin and one side as $x + y - 2 = 0$ is

(A) (–1, –1) (B) (2, 2) (C) (–2, –2) (D) (2, –2)

[**Hint**: Let ABC be the equilateral triangle with vertex A (h, k) and let D (α, β) be the point on BC. Then $\frac{2\alpha + h}{3} = 0 = \frac{2\beta + k}{3}$. Also $\alpha + \beta - 2 = 0$ and $\left(\frac{k-0}{h-0}\right) \times (-1) = -1$].

Fill in the blank in Exercises 42 to 47.

42. If a, b, c are in A.P., then the straight lines $ax + by + c = 0$ will always pass through ____.

43. The line which cuts off equal intercept from the axes and pass through the point (1, –2) is ____.

44. Equations of the lines through the point (3, 2) and making an angle of 45° with the line $x - 2y = 3$ are ____.

45. The points (3, 4) and (2, – 6) are situated on the ____ of the line $3x - 4y - 8 = 0$.

46. A point moves so that square of its distance from the point (3, –2) is numerically equal to its distance from the line $5x - 12y = 3$. The equation of its locus is ____.

47. Locus of the mid-points of the portion of the line $x \sin\theta + y \cos\theta = p$ intercepted between the axes is ____.

State whether the statements in Exercises 48 to 56 are true or false. Justify.

48. If the vertices of a triangle have integral coordinates, then the triangle can not be equilateral.

49. The points A (– 2, 1), B (0, 5), C (– 1, 2) are collinear.

50. Equation of the line passing through the point $(a \cos^3\theta, a \sin^3\theta)$ and perpendicular to the line $x \sec\theta + y \operatorname{cosec}\theta = a$ is $x \cos\theta - y \sin\theta = a \sin 2\theta$.

51. The straight line $5x + 4y = 0$ passes through the point of intersection of the straight lines $x + 2y - 10 = 0$ and $2x + y + 5 = 0$.

52. The vertex of an equilateral triangle is (2, 3) and the equation of the opposite side is $x + y = 2$. Then the other two sides are $y - 3 = (2 \pm \sqrt{3})(x - 2)$.

53. The equation of the line joining the point (3, 5) to the point of intersection of the lines $4x + y - 1 = 0$ and $7x - 3y - 35 = 0$ is equidistant from the points (0, 0) and (8, 34).

54. The line $\frac{x}{a}+\frac{y}{b}=1$ moves in such a way that $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$, where c is a constant. The locus of the foot of the perpendicular from the origin on the given line is $x^2 + y^2 = c^2$.

55. The lines $ax + 2y + 1 = 0$, $bx + 3y + 1 = 0$ and $cx + 4y + 1 = 0$ are concurrent if a, b, c are in G.P.

56. Line joining the points (3, – 4) and (– 2, 6) is perpendicular to the line joining the points (–3, 6) and (9, –18).

Match the questions given under Column C_1 with their appropriate answers given under the Column C_2 in Exercises 57 to 59.

57.

Column C_1	**Column C_2**
(a) The coordinates of the points P and Q on the line $x + 5y = 13$ which are at a distance of 2 units from the line $12x - 5y + 26 = 0$ are	(i) (3, 1), (–7, 11)
(b) The coordinates of the point on the line $x + y = 4$, which are at a unit distance from the line $4x + 3y - 10 = 0$ are	(ii) $\left(-\frac{1}{3},\frac{11}{3}\right),\left(\frac{4}{3},\frac{7}{3}\right)$
(c) The coordinates of the point on the line joining A (–2, 5) and B (3, 1) such that AP = PQ = QB are	(iii) $\left(1,\frac{12}{5}\right),\left(-3,\frac{16}{5}\right)$

58. The value of the λ, if the lines $(2x + 3y + 4) + \lambda\,(6x - y + 12) = 0$ are

Column C_1	**Column C_2**
(a) parallel to y-axis is	(i) $\lambda=-\frac{3}{4}$

(b)	perpendicular to $7x + y - 4 = 0$ is	(ii)	$\lambda = -\frac{1}{3}$
(c)	passes through (1, 2) is	(iii)	$\lambda = -\frac{17}{41}$
(d)	parallel to x axis is	(iv)	$\lambda = 3$

59. The equation of the line through the intersection of the lines $2x - 3y = 0$ and $4x - 5y = 2$ and

	Column C_1		**Column C_2**
(a)	through the point (2, 1) is	(i)	$2x - y = 4$
(b)	perpendicular to the line $x + 2y + 1 = 0$ is	(ii)	$x + y - 5 = 0$
(c)	parallel to the line $3x - 4y + 5 = 0$ is	(iii)	$x - y - 1 = 0$
(d)	equally inclined to the axes is	(iv)	$3x - 4y - 1 = 0$